January 2002

WELCOME

Mark Ortiz Automotive is a chassis consulting service primarily serving oval track and road racers. This newsletter is a free service intended to benefit racers and enthusiasts by offering useful insights into chassis engineering and answers to questions. Readers may mail questions to: 155 Wankel Dr., Kannapolis, NC 28083-8200; submit questions by phone at 704-933-8876; or submit questions by e-mail to: markortiz@vnet.net. Readers are invited to subscribe to this newsletter by e-mail. Just e-mail me and request to be added to the list.

SUSPENSION NATURAL FREQUENCIES

What suspension frequency should I be looking for in a formula car? Should the front and rear suspension have equal or similar frequency?

For readers less familiar with the subject, this question concerns the sprung mass natural frequency in ride, for the front and rear wheel pair systems. Natural frequency is a measure of how stiff the suspension is, which expresses stiffness in a way that is independent of the vehicle size or weight. That is, a given frequency feels comparably stiff whether the car is big and heavy or small and light, unlike a given spring rate or wheel rate. If the car is twice as heavy and the wheel rate is also twice as stiff, the natural frequency is the same.

A natural frequency is a vibration frequency at which a system will vibrate or oscillate when displaced from its static position, and released. For a simple system consisting of a mass supported by a spring:

$$f = .159\ (S/m)^{1/2}$$

where

f = natural frequency in Hertz (Hz), or oscillations per second.
S = spring rate, in Newtons/meter (N/m).
m = mass supported by the spring, in kilograms (Kg).

The quantity S/m is the inverse of m/S, which is the *static deflection* – the amount of spring compression at static condition. We may say that the natural frequency is inversely proportional to the square root of the static deflection. We may also say that the static deflection is inversely proportional to the spring rate, for a given mass. Therefore, natural frequency is proportional to the square root of spring rate, other factors held constant. So if we put springs with twice the rate on a car, the front and rear natural frequencies increase by a factor of the square root of 2.

The simple formula above is not really a very good approximation of an automobile suspension, especially a racing vehicle with stiff springs, stiff damping, and aerodynamic downforce. The tire is compliant, and its compliance is in series with the wheel rate. This makes the system softer, and the natural frequency lower, than if the tire were rigid. This is true for all cars, but it assumes particular significance when the suspension is stiff and the tire compliance is a large percentage of the total. If we know a spring rate for the tire, and the wheel rate from the suspension, the rate of the system is:

$1/S_c = 1/S_s + 1/S_t$, or $S_c = (S_s S_t) / (S_s + S_t)$

where

S_c = spring rate of the combination.
S_s = wheel rate of the suspension.
S_t = spring rate of the tire.

So if the tire is as compliant as the suspension, the frequency is only about .71 times what it would be if the tire were rigid.

Note that the above formulas assume a constant spring rate. In actual cars, we often have a rising wheel rate. This means that the frequency varies depending on suspension position. Some authors have suggested that natural frequency diminishes as downforce increases, because deflection increases. This is a misconception. Natural frequency is sensitive to the relationship between weight (mass times gravity) and effective wheel rate, and nothing else. This does not mean that rising-rate suspension is bad.

We have so far been talking about *undamped* natural frequency. Damping a system raises its natural frequency. To calculate a damped natural frequency, we need a constant spring rate, and a constant damping coefficient. In actual cars, damping coefficient is not constant, so any value we use for calculations has to be an approximation based on shock dyno outputs for velocities in the range we are trying to model. Stiff damping can raise frequency by as much as 30%, compared to the same system completely undamped.

As if this were not enough complexity, a suspension system will usually have different wheel rates when absorbing a one-wheel bump (combined roll and ride, or oppositional and synchronous motion) than when absorbing a two-wheel bump (pure ride or synchronous motion). The unsprung masses also have their own natural frequencies, much higher than the sprung mass frequencies, and these also vary depending on whether the wheels are moving synchronously or oppositionally.

In passenger cars, we have additional natural frequencies for masses flexibly attached to the main sprung mass, such as the engine on its mounts and the driver on a springy seat. These can be tuned to interfere with the ride motions of the sprung structure, and some useful ride damping can be achieved this way. Conversely, if these frequencies reinforce, ride quality will be adversely affected.

Passenger car engineers also look at “bounce” frequency (front and rear suspensions moving in the same direction, or basically in heave, approximating car behavior when both ends are excited together, as when traversing long humps and dips) and “pitch” frequency (car oscillating about a node near its middle, front and rear suspensions moving opposite directions, relevant to excitation by a short bump struck by first the front wheels and then the rears).

Despite all this, we can actually tell quite a bit about a car’s characteristics by just looking at the front and rear undamped frequencies, calculated using the front and rear portions of the sprung mass and the combined wheel rates for the front and rear wheel pairs. As previously noted, these will be inversely proportional to the square root of the static deflection for the front and rear. With asymmetrical cars, that would be an average of left and right static deflections, for the front and rear.

The equations given earlier are for frequency in Hertz, which is cycles or oscillations per second. It has also been customary, especially in countries using English units, to express suspension frequencies in oscillations or cycles per minute (opm or cpm). This figure will of course be 60 times the frequency in Hertz. It may also be calculated from static deflection in inches as follows:

$F = 188 / (x^{1/2})$

where

F = frequency in opm or cpm

x = static deflection in inches

Note that static deflection is calculated from the point where the spring is completely unloaded, which may be different from full droop on the suspension if the spring is loose or preloaded at full droop. When wheel rate is not constant, the only way to obtain valid static deflection for frequency calculation is to determine the instantaneous wheel rate at static ride height and divide by sprung weight.

Ranges of frequencies commonly found in different types of vehicles are:

Very soft passenger car: 0.5 to 0.8 Hz (30 to 50 opm)
More sporting passenger car: 1.0 to 1.3 Hz (60 to 80 opm)
Modern sports car: 1.1 to 1.5 Hz (70 to 90 opm)
Pavement race car with modest downforce: 1.5 to 2.0 Hz (90 to 120 opm)
Modern race car with ample downforce and ground effect: 5.0 Hz (300 opm) or more

As to the relationship between front and rear frequencies, the traditional answer from passenger car engineering is that the front static deflection should be about 30% greater than the rear. This would mean that the ratio of front frequency to rear frequency would be around .88. Such a relationship makes the front end and the rear end roughly go up and down together on the first bounce after a short disturbance, while allowing the car to ride long disturbances with a minimum of pitch. It also allows for some passengers and luggage in the rear.

This approach presents problems in rear-engine cars, however, and even in front-engine cars with aerodynamics that are sensitive to front ride height. The main difficulty is that front ride height changes too much in braking and forward acceleration. With tail-heavy cars, a very stiff front anti-roll bar is usually required if front roll resistance is to be adequate to assure proper load transfer allocation with soft front springs. In braking and under power, the front end will rise and fall dramatically, and if we have a front valance skirt, a front splitter, or a front wing, aerodynamic properties will be too sensitive to braking, power application, and disturbances from the track surface.

Consequently, race cars often have higher natural frequencies in front than in back. Front to rear frequency ratios may reach 1.5 or greater. This actually represents a return to the frequency relationships of the early 1930's, when cars had beam axles and longitudinal leaf springs, anti-roll bars were uncommon, and the front springs were spaced closer than rears for reasons of steering clearance. With these characteristics, and 50% or greater static rear percentage, the car will oversteer unless the front springs are a good deal stiffer than the rears.

It turns out that having the front frequency a lot higher than the rear is a good second choice in terms of ride, if it is impractical to make the front a bit softer. If you make the front and rear frequencies similar, or the rear a little softer than the front, the car pitches excessively on short bumps, although it rides long disturbances very nicely.

All of this is more important in softly damped passenger cars, and softly sprung and damped dirt cars, than in firmly damped pavement race cars. With firm damping, the first and second oscillations after a disturbance are not much of a concern, as the damping suppresses them in any case. Consequently, many race engineers pay little attention to frequency relationships unless the car exhibits ride motions bad enough to hurt lap time or make the driver complain.

Choice of frequencies then comes down to factors other than ride motion. There is no perfect set of springs for all tracks, as shown by the fact that almost everybody uses different combinations depending on where they're running. The most fundamental tradeoff is between ability to ride bumps (softer is better) and ability to limit ground clearance changes, reduce CG height, and control camber (stiffer is better). The bumpier the track, the softer and higher we have to run the car. The smoother the track, the more sensitive the car is to ground effects, the more sensitive the ground effects are to pitch and roll, and the more downforce the car generates, the stiffer we have to make the springs. On ovals, steeper banking calls for stiffer springs. In general, we can use softer springs if the car provides roll or pitch resistance from geometry or interconnective springs such as anti-roll bars. So we are faced with a complex compromise, which cannot be reduced to a simple formula, or even a simple rule about frequency relationships.

Finally, in production cars, we may stiffen up the springs just to crutch bad geometry or limit the antics of leaf-sprung axles. As Colin Chapman reportedly once said, "Any suspension will work if you don't let it."