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WELCOME

Mark Ortiz Automotive is a chassis consulting service primarily serving oval track and road racers. This newsletter is a free service intended to benefit racers and enthusiasts by offering useful insights into chassis engineering and answers to questions. Readers may mail questions to: 155 Wankel Dr., Kannapolis, NC 28083-8200; submit questions by phone at 704-933-8876; or submit questions by e-mail to: markortiz@vnet.net. Readers are invited to subscribe to this newsletter by e-mail. Just e-mail me and request to be added to the list.

PANTERA FRONT END REVISITED

The person who submitted last month's question about his Pantera's steering geometry informs me that the car does not have increasing anti-dive in droop, contrary to my speculation. The car has inner control arm axes that are very close to perfectly horizontal and longitudinal, meaning it has little or no anti-dive at any ride height.

In such a front end, the presence of substantial toe-in in droop, with minimal toe change in bump, suggests that (assuming front steer) the rack length is a little too long for the control arm layout – at least at the height the rack is mounted. This makes the tie rods a little too short, and produces a bump steer curve that we could call C-shaped, as opposed to J-shaped. But we can minimize bump steer in bump, and therefore on the loaded wheel when cornering, by raising the rack, or lowering the outer tie rod ends, relative to the setting that would give least instantaneous rate of toe change at static position. In other words, we can tilt the C-shaped curve. We can make the wheel toe out a little, and then in a little, through the bump range, at the expense of having it toe in a lot in extreme droop.

This is not an ideal situation, and it is doubtful that the builder is trying to get any benefits this way. Rather, this looks like a way to use a readily available rack that's a little longer than ideal, without getting roll oversteer.

HORSEPOWER AND TORQUE REVISITED

The author of the article on horsepower and torque on which I offered some comments last month points out that he was not necessarily assuming equal road speed in the cases he was comparing. I agree that it is possible for a car with lower horsepower to accelerate faster than one producing more horsepower, and to correspondingly have higher axle torque, if we are supposing that the car with less power is at a lower road speed. This is an unusual way to compare two cars, but the point does stand.

A corollary is that a given car accelerates slower and slower the faster it goes. Most of us know this already, but perhaps not everyone is aware that this would be true even if aerodynamic drag were absent or did not vary with speed. As road speed rises, a car has to use a numerically lower overall gear ratio to get a given engine rpm, and consequently torque multiplication diminishes.

Be this as it may, at any given road speed, other things being equal, more torque at the axle still implies more power at the engine, and does not necessarily imply more torque at the engine.

FRONT AND REAR CENTER OF GRAVITY?

I write to you in the hope that you may be able to help me with a few calculations regarding center of gravity. The situation is that I am chasing to restore the balance of our car (Australian Sports Sedan – 20B/RX7) once had. [The writer is referring to an early Mazda RX-7 with a later twin-rotor engine installed.] *The car's balance was disturbed once we fitted the 20B engine, adding about 200 lbs of weight. After chasing our tails for 4 years I decided to do something about it. I collected all of Carroll Smith's books, and Allan Staniforth to mention another, in order to gain a better understanding of racecar dynamics.*

Of interest at this time is a chapter in Allan Staniforth's book written by David Gould. The chapter has some nice formulae to determine lateral weight transfer and roll resistance. However, I am stuck at determining a few things that the formulae require.

The first problem I have is that he asks for the height of the CG of the front or rear unsprung mass (to calculate front and rear unsprung weight transfer). The only clue given to work this out is that the figure is usually similar to the radius of the tire. Is there another way of calculating these figures?

The second problem is similar: the equation to determine the mean center of gravity of the sprung mass. The author has mentioned here that to find the center of gravity at the front and rear axle lines can be an extremely difficult process and the preferred process is deduction from the known location of major items that comprise the sprung weight. Once again, would you know of any other techniques that can be used to determine the height of the center of gravity at the front and rear axle lines?

It is customary to model a car as a single sprung mass, flexibly connected to front and rear axles. Even if the suspension is independent, we approximate the independent systems at the front or rear to an equivalent single axle for modeling purposes. We thus have three bodies, each of which has a center of gravity (or center of mass). We may also regard the whole vehicle as a single mass, and this will also have a center of mass, which can be taken as a weighted average of the centers of mass of the three masses that comprise it. All three of these masses are assemblies of components, all of which have their own centers of mass, but we treat the car as three masses because these three are movable with respect to each other, for chassis analysis purposes.

For most designs, it works quite well to assume that the CG heights of the front and rear unsprung masses are at their respective hub heights, or tire loaded radii. This approximation is very accurate for the wheels and brake rotors. Other components may be higher or lower, and you can weigh them and estimate their individual CG locations, then calculate a CG location for the entire unsprung assembly, but for most cars this will come out very close to hub height.

I am amazed at the number of books that talk about front and rear **sprung** mass centers of gravity. The sprung mass is a relatively rigid single assembly, and it has one center of gravity (or center of mass), not a front and a rear one, not a string of them. The sprung mass is not two men in a horse suit, nor is it a train of flexibly connected bodies. Like any other body, it has one and only one point at which (or through which) a linear force can be applied from any angle without causing the mass to rotate in any direction, and that point is the center of mass. This point lies between the axles, at a height somewhere between 12 and 20 inches for most cars.

The best method to estimate the location of the center of gravity for the sprung mass, or the whole car, depends on whether we are dealing with a design or an actual car. If we are trying to model a car not yet built, we have to weigh as many purchased components as we can, estimate where their individual centers of mass are, estimate the weight and individual centers of mass for components we're designing, and then take moments in three planes about a convenient origin point to calculate the overall CG. One common choice for this origin point is a point centered between the front contact patch centers, at ground level. Another is a point centered laterally, and midway along the wheelbase, at ground level. Any point at all can work mathematically, but these are convenient. To estimate the overall vehicle CG location, we include the unsprung components. To estimate the sprung mass CG location, we omit the unsprung components.

For a car that's an assembled vehicle, sitting in our shop, we directly measure the whole car rather than taking it apart and weighing components. To find the lateral and longitudinal coordinates of the overall CG, we simply scale the car as we usually would when setting up for a race, and calculate the CG position in plan view from the left and rear percentages. If the car has, for example, 55% rear, then the overall CG is 55% of the wheelbase aft of the front axle line. If it has 51% left, then we find the overall CG's lateral position as follows:

1) In a top or plan view, construct lines connecting the contact patch centers on each side of the car.
2) Construct a line perpendicular to the vehicle centerline, through a point 55% of the wheelbase aft of the front axle.
3) Find the length of the line from step 2 between the lines from step 1.
4) Find the point on this line 51% of its length from the right end. This is the plan view location of the overall CG.

That's two thirds of our answer. We still need to find the height of the overall CG. This is the tricky part. The method usually described in chassis books involves elevating one end of the car, with the other end on scales, and noting the weight change at the scales. The distance of the CG above the unraised axle, the one on the scales, is given by: $h = [(\Delta W)(L)\sqrt{(L^2 - x^2)}] / (W)(x)$, where:

h = CG height above the axle on the scales, at static condition
W = total vehicle weight
ΔW = weight increase on the scales when opposite axle is raised

L = wheelbase length
x = height that axle not on scales is raised

Units of length for h, L, and x can be inches, feet, meters, or whatever, but must all match. Likewise, units of weight for W and ΔW must match.

One drawback to this method is that the overall CG of many race cars is not much above the axle lines, meaning that we are measuring a very small h. This means that accuracy becomes a problem. The smallest shift in fluids or other masses can noticeably affect the measurement, and the car may have a significantly different CG height if we drain all the oil and fuel than it has with a normal load of fluids. This may require a calculated correction. The end that's raised has to be raised a significant distance, generally at least 3 feet, and must be supported with the wheels free-rolling on perfectly level surfaces. This requires special stands and the means to hoist one end of the car that far. The suspension should be immobilized at normal ride height. The car may be rather precariously balanced when raised, and in many cars ground clearance of the front and rear overhangs will limit how far we can lift either end, unless we support the lower end on pedestals too.

A client of mine came up with an alternative method. It likewise requires immobilizing the suspension and draining the fuel and oil. He raises one side of the car, rather than one end. He has made large "shoes" of angle stock that cradle the two tires that remain on the ground. He uses a wrecking truck to lift the other side of the car. He raises it until the car balances on the edges of the "shoes". He leaves the hoist attached, so the car can't fall over, and gives the car just a little slack. He can then stand beside the car, move it gently by hand, and feel for the balance point. Once he finds the balance point, he measures the car's tilt.

Once that point is found, he knows that the overall CG lies in the vertical plane passing through the "shoe" edges. This we might call the balance plane. When the car is sitting normally on its wheels, this plane assumes an angle from vertical equal to the tilt of the car when the balance plane was found. He knows the plan-view position of the CG from scaling the car during setup, as described above. So the CG lies on a vertical line that appears as a point in plan view. He just finds where the balance plane intersects that vertical line, and that's the overall CG.

Once we have the overall CG location, we can calculate sprung mass CG if we have weights and estimated CG heights for the unsprung masses. For cars with independent suspension all around, the sprung mass CG will be at approximately the same lateral and longitudinal position as the overall CG, and a little higher. For cars with live rear axles, the sprung mass CG will be a bit forward of the overall CG, and a little higher.

If all this seems intimidating, be aware that you can do calculation exercises, and learn a lot about vehicle dynamics, using assumed values for the CG locations. You can also determine the longitudinal and transverse location of the overall CG by measurement, and assume only the height. For your RX-7, assumed heights of 16 inches for the overall CG, 17.5 inches for the sprung mass CG, and tire loaded radius for the front and rear unsprung mass CG's are probably pretty realistic.

As a general rule, to re-balance the cornering of a car that has received a larger engine, you need to lighten whatever you can in the front, move whatever you can to the rear, increase the share of the roll resistance at the rear, and adjust the brake bias for more front.